

George Polya's Contribution to Lakatos Philosophy of Mathematics

Steven Meyer

Tachyon Design Automation, Boston, MA USE
smeyer@td1.com

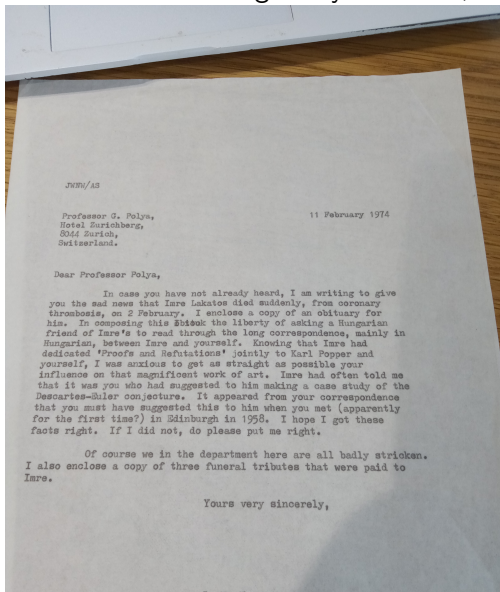
Presented November 3, 2022 at Lakatos Centennial Conference
London

Introduction

- The best way to understand the importance of George Polya's influence on Imre Lakatos philosophy of mathematics is to read the correspondence.
- I will start with the questions John Watkins asked Polya after Lakatos death from the John Watkins Archive. Then show the correspondence between Polya and Lakatos from 1958 to 1964 from the Lakatos Archive.
- I have included a letter from Paul Feyerabend to Lakatos in 1967 on Feyerabend's philosophy of physics and the "Defense of Classical Physics" paper that Lakatos and Alan Musgrave helped with. The paper uses Polya's 1920 central limit theorem of probability.
- I will upload these slides with the pdfs of the original images and my typed in text to my web site. Go to www.tdl.com/~smeyer. You will see a page titled "Steve Meyer's Research Web Page". Select the blue link under item "slides from Lakatos Centenary Conference 2022".

Watkins first letter image

Letter John Watkins to George Polya Feb. 11, 1974 from Watkins



Watkins first letter text

- Professor G. Polya, Hotel Zurichberg, 8044 Zurich, Switzerland.
- Dear Professor Polya,
- In case you have not already heard, I am writing to give you the sad news that Imre Lakatos died suddenly, from coronary thrombosis, on 2 February. I enclose a copy of the obituary for him. In composing this [I took] the liberty of asking a Hungarian friend of Imre's to read through the long correspondence, mainly in Hungarian, between Imre and yourself. Knowing that Imre had dedicated 'Proofs and Refutation' jointly to Karl Popper and yourself, I was anxious to get as straight as possible your influence on that magnificent work of art. Imre had often told me

Watkins first letter continued ...

- that it was you who had suggested to him making a case study of the Descartes-Euler conjecture. It appeared from your correspondence that you must have suggested this to him when you met (apparently or the first time?) in Edinburgh in 1958. I hope I these facts right. If I did not, do please put me right.
- Of course we in the department here are all badly stricken. I also enclose a copy of three funeral tributes that were paid to Imre.

Polya reply to Watkins letter front image

Letter George Polya to JW March 16, 1974 front

Palo Alto, 16 March 1974

Dear Professor Watkins,

I am very grateful for your letter of 11 February which, forwarded from Zurich, reached me today.

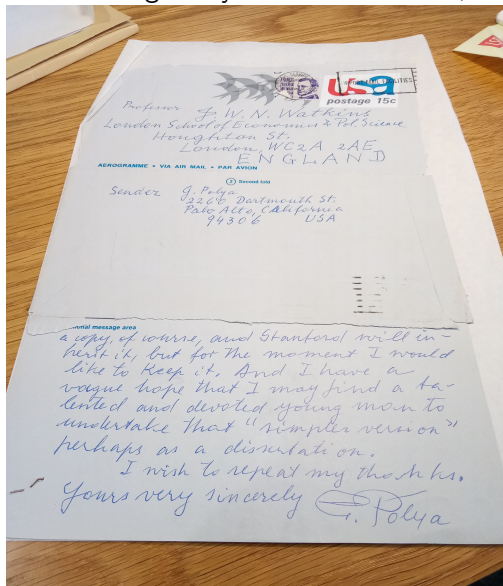
I heard the sad news about Imre already - a Cambridge friend sent me the obituary from the Times, which, as I see from your letter, you have written. I am grateful for this obituary; it seems to me very good. Especially the facts about my connection with Imre and his dissertation are completely accurate.

I told Imre, and he agreed to some extent, that his "Proofs and Refutations" are too brilliant, the dialogue with the major part of the Greek alphabetists involved, he should publish it again, but followed by a simpler and more direct version - it would have a broader, and possibly deeper, influence. May I ask you: Is there, among his papers, at least a beginning of that simpler version?

Are there, among his papers, a few reprints of "Proofs and Refutations"? If there are, could I have one for the Library of Stanford? I have

Polya reply to Watkins letter back image

Letter George Polya to JW March 16, 1974 back



Polya reply text

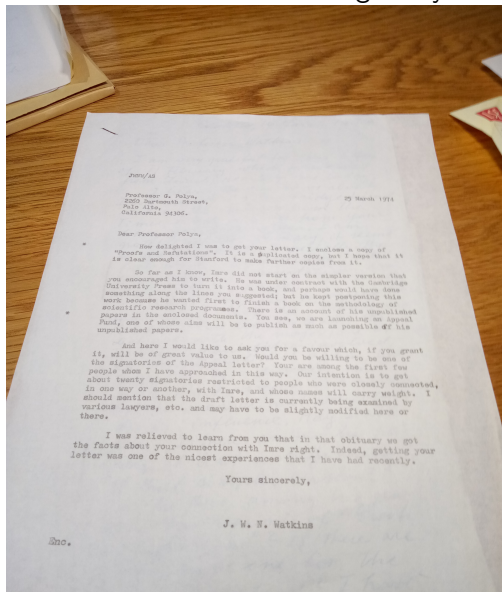
- Palo Alto 16 March 1974 Dear Professor Watkins
- I am very grateful for your letter of 11 February which, forwarded from Zurich reached me today.
- I heard the sad news about Imre already – a Cambridge friend sent me the obituary from the Times, which, as I see from your letter, you have written. I am grateful for this obituary, it seems to me very good. Especially, the facts about my connection with Imre and his dissertation are completely accurate.

Polya reply text continued ...

- I told Imre, and he agreed to some extent, that his “Proofs and Refutations” are too brilliant, the dialogue with the major part of the Greek alphabet is too involved, or he should publish it again, or he should publish it again, or followed by a simpler and more direction version – it would have a broader, and possibly deeper, influence. May I ask you : Is there, among his papers at least a beginning of that simpler version?
- Are there among his papers, a few reprints of “Proofs and Refutations”? If there are, could I have one for the library at Stanford? I have
- a copy, of course, and Stanford will inherit it, but for the moment I would like to keep it. And I have a vague hope that I may find a talented and devoted young man to undertake that “that simpler” version perhaps as a dissertation.
- I wish to repeat my thanks.
- Yours very sincerely G Polya

Watkins reply to Polya letter image

Letter John Watkins to George Polya March 25, 1974



John Watkins reply text

- Professor G. Polya, 2260 Dartmouth Street, Palo Alto, California 94306.
- Dear Professor Polya,
- How delighted I was to get your letter. I enclose a copy of “Proofs and Refutations”. It is a duplicated copy, but I hope that it is clear enough for Stanford to make further copies from it.
- So far as I know, Imre did not start on the simpler version that you encouraged him to write. He was under contract with the Cambridge University Press to turn it into a book, and perhaps would have done something along the line you suggested; but he kept postponing this work because he wanted first to finish a book on the methodology of scientific research programmes. You see, we are launching an Appeal Fund, one of whose aims will be to publish as much as possible of his unpublished papers.

John Watkins reply continued ...

- And here I would like to ask you for a favour which, if you grant it, will be of great value to us. Would you be willing to be one of the signatories of the Appeal letter? You are among the first people whom I have approached in this way. Our intention is to get about twenty signatories restricted to people who were closely connected, in one way or another, with Imre, and whose names will carry weight. I should mention that the draft letter is currently being examined by various lawyers, etc. and may to be slightly modified here or there.
- I was relieved to learn from you that in that obituary we got the facts about your connection Imre right. Indeed, getting your letter was one of the nicest experiences that I have had recently.
- Yours sincerely, J. W. N. Watkins

Polya reply to Watkins Lakatos appeal request answer

Letter George Polya to John Watkins April 27, 1974

Palo Alto, 26 April 1974

Dear Professor Watkins,

Many thanks for your very kind letter.

I am particularly grateful for the copy of "Proofs and Refutations". It is a great loss that Imre did not turn it into a book. There is a gap that cannot be filled. Yet still I shall try to find somebody who can do at least a little: write a simplified, more accessible version of P. & R.

Many thanks also for the typescript of the unpublished paper "A Renaissance of Empiricism etc". I looked at it repeatedly (hence the delay of this letter), trying to find something to ~~fill~~ ^{fill} that gap - but no success.

Of course, I shall be delighted to ^{sign the} ~~lighted to~~ letter for the "Imre Lakatos Appeal Fund".

I wish to repeat my thanks for your very kind letter.

Sincerely

Polya reply text

- Palo Alto 20 April 1974 Dear Professor Watkins
- Many thanks for your very kind letter.
- I am particularly grateful for the copy of “Proofs and Refutations.” It is a great loss that Imre did not turn it into a book. There is a gap that cannot be filled. Yet still I shall try to find somebody who can do at least a little : write a simplified, more accessible versions of P.&R.
- Many thanks also for the type script of the unpublished paper “A Renaissance of Empiricism etc” I looked at it repeatedly (hence the delay of this letter) trying to find something to fill that gap - but no success.
- Of course, I should be delighted to sign the letter for “the Lakatos Appeal Fund”.
- Yours sincerely G. Polya

Original Polya letter to Lakatos in Hungarian Lakatos Archive

Letter George Polya to Imre Lakatos Dec. 6, 1958 - Lakatos

Translation of item 226.

POLYA to LAKATOS

Zurich 7
Karlhaus Zurichberg

8.12.1958

My dear friend,

Many thanks for your letter of 30th November, also - with ugly delay, for your postcard from the beginning of October. Let us start with the question to which the answer is simple: I shall be staying here until 27th December, when I shall leave - via Paris, Garmouth and New York but without stopping - for California. My address from cca. 10th January: Stanford University, Stanford, California, U.S.A.

I am very glad that you are studying with interest those things which you feel belong to your topic; One must follow one's feelings (with a certain degree of criticism and scepticism). You will return to Euler's theorem "when the spirit moves you" and this is how it should be.

Gomathi and Bernays are giving a joint seminar here. The subject for this term has been induction ("physical" induction) and I have taken part in the discussion industriously - I have realised that what I had said about this in "Maths. and Plausible Reasoning" needs amplification.

It would be interesting to discuss this and other things with you at Stanford. From among the logicians at Stanford (the philosophers don't count) the person who could do most for arranging your visit to Stanford is Suppes (Brathwaite knows him). Then there is the Institute for Advanced Study in the Behavioural Sciences (Ford Foundation) in Stanford, through which one could arrange something. Unfortunately my connection with this Institute (popular name "think factory") is ≈ 0 .

True, I travel a great deal from Stanford (e.g. I shall lecture at Berkeley during the first term of 1959, so I shall spend 2-3 days there weekly). I want to hope that I shall survive that, too, and that there will be times when I shall not be too exhausted after my journeys here to Palo Alto.

Write to me if you think that I can help a little bit. With all good wishes,

Original Polya 1958 to Lakatos text

- Zurich 7 Kurhaus Zurichberg 8.12.1958 My dear friend,
- Many thanks for your letter of 30th November, also – with ugly delay, for your postcard from the beginning of October. Let us start with the question to which the answer is simple; I shall be staying here until 27th December, when I shall leave – via Paris, Cherbourg and New York but without stopping – for California. My address from ccu. 10th January: Stanford.
- I am very glad that you are studying with interest those things which you feel belong to your topic; One must follow one's feelings (with a certain degree of criticism and scepticism). You will return to Euler's theorem “when the spirit moves you” and this is how it should be.
- Gonseth and Bernays are giving a joint seminar here. The subject for this term has been induction (“physical” induction) and I have taken part in the discussion industriously – I have realized that what I had said about this in “Maths. and Plausible Reasoning”

Polya 1958 continued ...

- needs amplification. It would be interesting to discuss this and other things with you at Stanford. From among the logicians at Stanford (the philosophers don't count) the person who could do most for arranging your visit to Stanford is Suppes (Braithwaite know him). Then there is the Institute for Advanced Study in the Behavioural Sciences (Ford Foundation) in Stanford, through which one could arrange something. Unfortunately my connection with this Institute (popular name: "think factory") is $= 0$.
- True, I travel a great deal from Stanford (e.g. I shall lecture at Berkeley during the first term of 1959, so I shall spend 2-3 days there weekly). I want to hope that I shall survive that, too, and that there will be times I shall be too exhausted after my journeys home to Palo Alto.
- Write to me if you think that I can help a little bit. With all good wishes,
- Gy. Polya

Lakatos to Polya Szabo discussed

Letter Imre Lakatos to George Polya July 20, 1965 Szabo Lakatos

Lakatos file 12-9, item 235

20th July 1965

Dear Professor Polya,

Thank God the conference which I have been organising during the last few months is now over and I can breathe again and, hopefully, get down to my work. It will take weeks to get back into the atmosphere of it. Never a conference again in my life!

Of course socially the meeting was quite pleasant and there were lots of interesting and nice people and I very much wished you had been here. Larmagne was one of the most brilliant contributors. He was really at the top of his form. One of the last successes of the conference by the way was my Hungarian friend Szabo, who delivered a brilliant lecture on Greek mathematics. I am now going to try to arrange for him a fellowship at Stanford - I have already spoken to Suppes and I am only waiting for the list of his publications to be sent to me and then with Forster and William Rouse and Larmagne we shall try to do something. I don't know whether you know his work; if so could you help?

I myself am planning again to go to California for a short time to learn some logic. This time I hope to come to Berkeley which will be near to you and so I hope that I shall have the opportunity of profiting by your help and advice.

I looked at Mary Hesse's book Models and Analogies. The Black, who was just now at Stanford and whom you may remember (he was at my lecture), also wrote a book in 1962 on Models and Analogies. Neither of them mentions you although Mary Hesse has a detailed list of references from which she quotes you. Frankly I don't think you should care too much: these, especially Mary Hesse, are illiterate philosophers without real standards. However I think this should be put right somehow and I am now going to put a research student of mine to work on this and perhaps a note will come out of it comparing your work on models and analogies with recent philosophical work on this subject. This will at least put the record right.

On your suggestion I thought I would try to do something about

Lakatos 12 file 9 item 235 Szabo Greek philosophy influence

- 20th July 1965, Dear Professor Polya,
- Thank God the conference which I have been organizing during the last few months is now over and I can breathe again and, hopefully, get down to my work. It will take weeks to get back into the atmosphere of it. Never a conference again in my life!
- Otherwise socially the meeting was quite pleasant and there were lots of interesting and nice people and I very much wished you had been here. Bernays was one of the most brilliant contributors. He was really at the top of his form. One of the best successes of the conference by the way was my Hungarian friend Szabo, who delivered a brilliant lecture on Greek mathematics. I am not going to try to arrange for him a fellowship at Stanford - I have already spoken to Suppes and I am only

Lakatos Szabo Greek philosophy continued ...

- waiting for the list of his publications to be sent to me and then with Popper and William Kneale and Bernays we shall try to do something. I don't know whether you know his work; If so could you help?
- I myself am planning again to go to California for a short time to learn some logic. This time I hope to come to Berkeley which will be near to you and so I hope that I shall have the opportunity of profiting by your help and advice.
- I looked at Mary Hesse's book Models and analogies. Max Black, who also wrote a book in 1967 on Models and methodology. Neither of them mention you although Mary Hesse has a detailed list of references from which she omits you. Frankly I don't think you should care too much:

Lakatos Szabo Greek philosophy second continued ...

- these, especially, Mary Hesse, are illiterate philosophers without real standards. However I think this should be put right somehow and I am now going to put a research student of mine to work on this and perhaps a note will come out of it comparing your work on models and analogies with recent philosophical work on this subject. This will at least put the record right.
- on Your suggestion I thought I would like to do something about [connection between heuristics and epistemology].
- ... second page of letter discussing people omitted

Polya reaction and criticism of P&R from Lakatos Archive

Letter George Polya Polya to July 20, 1965 - Szabo Lakatos

Translation of item 296

POLYA to LAKATOS

Kurt H. Zurichberg
Zurich 44
15.11.1965

Dear Lakatos,

At last I have had a relatively peaceful week: relative freedom of proceedings, translations (Math. Discovery into German; an earlier, long article of mine into English) etc. I have read "Proofs and Refutations" (P & R) twice from beginning to end (Part IV or V, handwritten unclear: Trans. I was new to me). I read it slowly, with all the concentration I am capable of. Then I wanted to write a long letter to you. I realized, however, in a fairly short time, that only a considerably shorter letter would have any chance of being written if I wanted to finish it in finite time. I had written three pages of it. I could not continue it; afterwards I could not continue it either; now I have torn those three pages into pieces and shall be writing a very short letter, and an untidy one as well - otherwise I shall never finish it.

My main impression: Do not attempt to keep correcting it - put P & R into a book as it is, apart from possible minor corrections of details. It is true, that P & R is, in many respects, confusing: if the reader tries to attach the different personalities to the various Greek letters which stand for them; if he tries further, to distinguish the numerous new terms from each other (on almost every page up jumps a new term) - then he would get dazed. At least this is the experience of the reader who is writing this letter. But, the article is interesting, amusing, humorous and, most of all "surprising", "stimulating" as it is: the elimination of each secondary disadvantage could be done only at the cost of losing an advantage of the first order - this is my strong impression. Therefore: quieta non movere. It would be possible to consider repeating the most essential points of P & R in a later chapter with fewer anecdotes and more calm - yes, think of it, consider it.

I can see fairly clearly how P & R relates to my own work. The basic difference is: I myself would hardly be able to say anything on "epistemology" which would deserve the attention of the public. Had I been able to say anything about it, even then I would have refrained from it: it is difficult enough to have the public accept heuristics, and I would not have wanted to make this even more difficult by combining it with other controversial things. The main point of P & R is, at least according to me, to call attention to the possible

(contd.)

Polya reaction to P&R Lakatos 12 file 9 item 236

- Kurhaus Zurichberg Zurich 44 15.11.65 Dear Lakatos,
- At least I have had a relatively peaceful week: relative freedom of proofreading, translations (Math. Discovery into German; an earlier, long article of mine into English) etc. I have read "Proofs and Refutations" (P & R) twice from beginning to end (Part IV [or V, handwriting unclear: Trans.] was new to me), I read it slowly, with all the concentration I am capable of. Then I wanted to write a long letter to you. I realized, however, in a fairly short time, that only a considerably shorter letter would have any chance of being written if I wanted to finish it in finite time. I had written three pages of it. I could not continue it; afterwards I could not continue it either; now I have torn those three pages into pieces and shall be writing a very short letter, and a untidy one as well – otherwise I shall never finish it.

Polya reaction to P&R continued ...

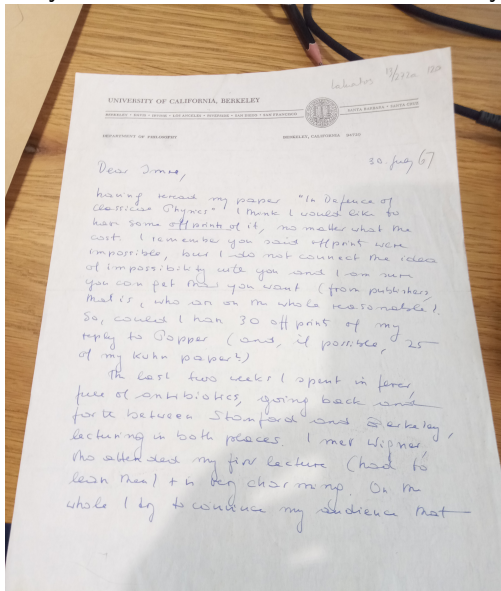
- My main impression: Do not attempt to keep correcting it – put P & R book as it is, apart from possible minor corrections of details. It is true, that P & R is, in many respects, confusing: if the reader tries to attach the different personalities to the various Greek letters which stand for them; if he tries further, to distinguish the numerous new terms from each other (on almost every page up jumps a new term) – then he would get dazed.
- At least this is the experience of the reader who is writing this letter. But, the article is interesting, amusing, humourous and, most of all “anregend”, “stimulating” as it is: the elimination of each secondary disadvantage could be done only at the cost of losing an advantage of the first order – this is my strong impression. Therefore: *quieta non movere*. It would be possible to consider repeating the most essential points of P & R in a later chapter with fewer anecdotes and more calm - yes, think of it, consider it.

Polya reaction to P&R continued ...

- I can see fairly clearly how P & R relates to my own work. The basic difference is: I myself would hardly be able to say anything on “epistemology” which would deserve the attention of the public. Had I been able to say anything about it, even then I would have refrained from it: it is difficult enough to have the public accept heuristics, and I would not have wanted to make this even more difficult by combining it with other controversial things. The main point of P & R is, at least according to me, to call attention to the possible
- connection between heuristics and epistemology. It makes a number of points about pure heuristics as well, which I have not seen so clearly, and at any rate, have not said.

Feyerabend to Lakatos letter on Classical Physics paper

Letter Feyerabend to Lakatos on Classical Physics paper July 30,



page 2 of letter ...

Hilbert and von Neumann ruined
 physics. And apparently no one
 is prepared to swallow, despite
 my splendid argument (that must
 be said: the formalism is my guest, so
 can now move in the formalism what
 before it accepted as the basis of a
 quest, but as a result the correlation
 to experience becomes as loose as
 we are sometimes almost stirring
 metaphysics). Interspersed in my lectures
 or quantum theory are wicker remarks
 and argument against philosophy of
 science in the Supper fashion. I might
 enjoy myself more were it not for the
 fact that my chewing muscles are in a
 spasm + swollen and I can hardly open
 my mouth (this is the reason for fever +
 chills last week 4 weeks).

Well, all the best and I hope
 to have my mouth in control when I meet
 you again.

Dear

P.S.: Do you want me to send
 a booklet for the
 two or three weeks, about
 the book?
 I think I want
 to see it
 LSD
 R.

Feyerabend to Lakatos on Classical physics 13 file 272a item 120

- Dear Imre, 30 July 67
- having reread my paper “In Defence of Classical Physics” I think I would like to have some off prints of it, no matter what the cost. I remember you said offprints were impossible, but I do not connect the idea of impossibility until you and I am sure you compel mss you want (from publishers that is, who are on the whole reasonable). So, could I have 30 offprints of my reply to Popper (and, if possible, 25 of my Kuhn paper?)
- The last two weeks I spent in fever, free of antibiotics, going back and forth between Stanford and Berkeley, lecturing in both places. I met Wigner, who attended my first lecture (had to leave then) & is very charming. On the whole I try to convince my audience

Feyerabend to Lakatos on Classical physics continued ...

- Hilbert and von Neumann ruined physics which apparently no one is prepared to swallow, despite my splendid arguments (main point, the formally ms are very [gorsist?], we can now prove in the formalism what before we accepted on the basis of a guess, but is the result the correlation to experience becomes so loose that we are sometimes almost [showing?] metaphysics).
- Interspersed in my lecture on quantum theory are wicked remarks and arguments against philosophy in the Suppes fashion. I might enjoy myself more were it not for the fact that my chewing muscles are in spasm & swollen and I can hardly open my mouth (this is the reason for fever will last more than 4 weeks). Well, [hope?] me best and I hope to open my mouth in comfort when I meet you again.
- P.S. Do you want me to send a book list for my lecture at LSE to recommend two or three books mainly, [something] no LSD or CR.

The Argument

1. Polya hoped second generation Popperians would help develop his heuristics research programme just as Max Planck and Albert Einstein hoped the Vienna Circle and Berlin Circle philosophers such as Moritz Schlick, Hans Reichenbach and Karl Popper would develop philosophy of physics.
2. Polya became more skeptical of Hilbert's formal logics program through his career. He hoped Lakatos' philosophy would help his programme.
3. I believe Lakatos quasi-empirical philosophy of mathematics (Polya heuristics) is modern and still progressive. Argument discusses Feyerabend's criticism of formalism in physics and gives modern examples.
4. My argument is effectively testifying from my experience as a Stanford undergraduate. Also, I shared an office in the UCB CS department with Diane McIntyre who had been in a mathematics teaching education program run by Polya.

Approximate Polya heuristics development time line

Other mathematicians such as Paul Finsler and Paul Bernays were involved in developing Polya's heuristic research programme.

- 1920 Polya named and introduced the central limit theorem of probability. It was based on work of Abraham de Moivre from 1733 according to Wikipedia.
- 1926 Paul Finsler criticizes the Hilbert formalization research program showing inconsistency and incompleteness of logic outside the Hilbert program without reference to any particular logical system.
- 1945 Polya publishes "How to Solve It". Also publishes "Mathematics and Plausible Reasoning".
- 1950s Polya helped a fellow Hungarian philosopher of science with a thesis topic idea. Polya seemingly felt his quasi-empirical mathematics research programme was not understood and needed assistance from philosophers who could discuss method. Polya encouraged and taught Lakatos.

Approximate Polya development continued ...

- In 1969 Finsler defined a continuum for which the continuum hypothesis is true contradicting Dana Scott's proof. Paul Bernay's publishes a paper discussing Finsler's proof.
- Late 1960s. Lakatos and other philosophers of science studied mathematical logic as induction. Probably easiest to understand expression of Polya's programme is Feyerabend's defense of classical physics in (*Feyerabend Volume 4, part 1, section 14*).
- Program ends as Polya ages and Lakatos dies in 1974. Polya explains his hopes for the progress Lakatos would have made in correspondence with John Watkins mid 1970s (see above).

Some Quotations

- In the introduction to “Proofs and Refutations” editors John Worrall and Elie Zahar write **[some mathematicians believe] while the method of ‘proof-analysis’ described by Lakatos may be applicable to the study of polyhedra, ..., it may be inapplicatable to ‘real’ mathematics.** I disagree with this statement.
- In “For and Against Method” Paul Feyerabend writes **[I] will criticize all theoreticians of science. By “all” I mean of course only those who count, viz. Popper, Kuhn, Lakatos (the anti-kuhn Lakatos and not the anti-Carnap and Karl (Popper) Lakatos).** p. 120

Three examples show why quasi-empirical mathematics is modern

1. In the above 1967 Feyerabend letter on physics to Lakatos, Feyerabend says von Neumann logic ruined physics. Feyerabend did not know that in the early 1950s, von Neumann agreed quantum logic was wrong.
2. An example from modern algorithmic quasi-empiricism in mathematics. The Dedekind cut definition of real numbers is not the same as the Cantor definition because the Cantor definition requires an equivalence class computation.
3. There are more than one possible definition of the unknown state in three value logic. Which is best is a quasi-empirical question.