George Polya's Contribution to Lakatos Philosophy of Mathematics

(extended abstract)

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Short Abstract: This paper argues that George Polya's influence on Imre Lakatos quasi empirical philosophy of mathematics (called Heuristic by Polya) is more significant that currently believed. Argument is based on mid 1970s correspondence between John Waktins and Polya. Polya was hoping that Lakatos would write an easier to understand Proofs and Refutations. to increase understanding of his heuristic research programme. The paper discusses the development of Polya's heuristic methodology of mathematics that expressed his skepticism toward Hilbert's logicism program. Arpad Szabo's influence on Lakatos Greek philosophy of mathematics is not discussed.

1. Introduction

It was thought until John Watkin's correspondence with George Polya was published in Watkin's archive (Watkins[2010]) that Polya suggested the idea for Imre Lakatos thesis, but following the suggestion there was little other communication between the two. The belief was based on the dearth of correspondence between the two in the Lakatos Archive (Lakatos[2002]). Appad Szabo influenced Lakatos on Greek mathematics. This different influence is not considered in this paper. See Mate[2006] for example. Much of the correspondence between Feyerabend and Lakatos in Lakatos[1999] involves Greek mathematics and science.

I believe that in fact Polya had become convinced that Hilbert's logicism program (Zach[2019]) was incorrect as early as the 1920s. This led to Polya's development of heuristic mathematics. Polya was not a philosopher so his writings involved quasi-empirical mathematical solution methods. He had been looking for philosophers who could better explain and develop the philosophical and historical aspects of the method. Imre Lakatos' need of a thesis topic provided the opportunity. Polya's anti formalist philosophy of mathematics research programme ended with Lakatos death. The Polya Watkins correspondence in the mid 1970s shows Polya's disappointment that his hoped for an easier to understand *Proofs and Refutations* (Lakatos[1976]) would not be written.

Polya had been developing anti-formalist mathematics methods starting in the 1920s at ETH in Zurich that continued when Polya moved to Stanford University in 1940. Polya called his mathematics "heuristic". Lakatos called it "quasi empirical". In Lakatos, Polya was relying on philosophers to continue methodological study just as Albert Einstein and Max Planck relied on the Vienna circle to continue methodological study of modern physics. Moritz Schlick was a student of Planck and Hans Reichenbach was influenced by Einstein. Schlick's paper intended to be presented at the 2nd International Congress on the Unity of Science in 1936 on limitations of possibility of knowledge in quantum mechanics (Bacciagalupi[2020]) is an example of philosophy of physics contributing to physics research.

Polya became skeptical in the 1920s probably influenced by Paul Finsler's 1926 proof of incompleteness and undecidability that criticized the Hilbert formalist logicist research from outside the programme ("Formal Proofs and Decidability", pp. 50-56 in Finsler[1996]). I claim Godel's more widely accepted proof of the incompleteness and inconsistency of Russell's logic is internal criticism of Hilbert's programme. Polya thought algorithmically so he developed and published his quasi empirical theory in *How to Solve it.* (Polya[1945]).

In 1969 Finsler at ETH published a proof and a set of axioms defining a continuum

hypothesis for which the conjecture is true different from previous definitions and proofs that it is independent of the choice of Zermelo Frankel set theory axioms (Finsler[1969]). Polya and other mathematicians felt that their quasi-empirical method was not being understood and needed philosophical help just as the founders of modern physics needed philosophical assistance. Finsler's proof has still not been translated from German and Hilbert's formalist precision is almost universally accepted. In his paper on tension between physics and mathematics (Redei[2020]), Miklos Redei assumes mathematical method follows Hilbert's programme. In the abstract he writes: "*An attempt is made to explain why mathematical precision is typically not welcome in physics.*"

I was an undergraduate student at Stanford in the late 1960s. The mathematics curriculum was influenced by Polya. The next section of this paper explains Polya's skepticism toward probability by examining in detail the central limit theorem that was reintroduced by Polya, and taught to Stanford undergraduates and studied by Lakatos and Paul Feyerabend.

In the late 1960s, there were two competing research programs at Stanford. There was the Patrick Suppes logicism program and the Polya quasi-empiricism program. Adherents of each programme did not communicate with the exception of Jaakko Hintikka who communicated with both groups. See for example, Can Baskett's (Baskent[2016]) argument that Hintikka's interrogative approach and Lakatos quasi-empiricism are similar. Since Hintikka developed logics, he also was a part of Suppes' program.

I attribute the quasi-empirical research program to Polya. There were other mathematicians who participated including at least Paul Finsler and Paul Bernays. Here is a rough time line of its development.

1. 1920 Polya named and introduced the central limit theorem based on work of Abraham de Moivre from 1733 according to Wikipedia.
2. 1926 Paul Finsler criticizes the Hilbert formalization research program showing inconsistency and incompleteness of logic outside the Hilbert program with reference to any particular logical system (Finsler[<ref>]).
3. 1945 Polya publishes *How to Solve It.* (Polya[1945]). Also publishes *Mathematics and Plausible Reasoning* volumes.
4. 1950s Polya helped philosopher of science with a thesis topic idea. Polya seemingly felt his quasi-empirical mathematics research programme was not being understood and needed assistance from philosophers who could debate methods. Polya encouraged and taught Lakatos (Watkins[2010]).
5. Finsler defined a continuum for which the continuum hypothesis is true contradicting Dana Scott's proof (Finsler[1969]). Paul Bernay's publishes a paper discussing Finsler's proof (Bernays[1971]).
6. Late 1960s. Lakatos and other philosophers of science studied mathematical logic as inductivism. Probably easiest to understand expression of Polya's program is Feyerabend's defense of classical physics (Feyerabend[1970]).
7. Program ends as Polya ages and Lakatos dies in 1974. Polya explains his hopes for work Lakatos would have accomplished in correspondence with John Watkins mid 1970s (Watkins[2010]).

1.1 Polya's 1930 Central Limit in Probability Theorem

In 1930, Polya introduced the central limit theorem. It was taught to Stanford second year

calculus students in a section of Tom Apostol's calculus book volume 2, pp. 152-154. I remember it being taught in a way to discourage students from taking probability theory too seriously. This is also the main argument of Feyerabend paper "*In Defense of Classical Physics*." (Feyerabend[1981]) See Lakatos[1999] for correspondence on Lakatos contribution to the paper.

2. Referencs

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