Modern Mathematics and Physics are Quasi-empircial in the Same Sense
(Extended Abstract)

A new theory of scientific research programmes developed during the 20th century by Lakatos, Feyerabend and Kuhn (LFK) has become widely accepted by theoretical physicists and mathematicians.[1] Lakatos' and Feyerabend's philosophy applied to both mathematics and physical sciences. Except Kuhn's *The Structure of Scientific Revolutions* applied to physical sciences only. The Lakatos-Feyerabend-Kuhn (LFK) research programme based philosophy identified a number of crucial aspects of science. The properties are: view science as competing research programmes, require empirical testability (quasi-empirical for math) and allow the existence of anomalies and scientific revolutions. An important prescriptive part of LFK philosophy is the need for research programme (theory or mathematical programme) proliferation. LFK believed in the importance of research programme competition so strongly that they proliferated three similar but competing philosophical theories that focused on different aspects of science.[2]

This paper argues that modern theoretical physics and mathematics are quasi-empirical.[3] Therefore mathematics is not special in sense of being unique. The "specialness" of both physics and mathematics compared to all other endeavors is not considered. The methodological similarity is shown by analyzing examples of quasi-empirical research programme disagreements and problems in modern mathematics. It is possible that Smolin's Kuhnian crisis in theoretical physics is caused by anomalies in mathematics. Obviously, once a mathematical theory is axiomatized, problem solving is no longer empirical, but although axiomatized physical theories are rare, the same non-empiricism applies to such theories. In his classic Quantum Mechanics text book Leonard Schiff states that for quantum mechanics, the formalism came first only later to be followed by its interpretation in physical terms (Schiff, L. *Quantum Mechanics*, 1949, p. 1).

Testing and progress in math and physics are both dependent on phenomenological interpretations. One simple physical example due to Andrew Pickering is that a theory of particle interactions is needed to interpret cloud chamber tracks ('separating the wheat from the chaff').[4] Smolin's criticism of string theory is based on the phenomenological nature of string theory as a mathematical meta theory. A carefully studied example from mathematical logic is Finsler's rejection of Tarski style meta-mathematics. Modern axiomatization requires belief in meta-mathematical concepts that Finsler rejected.[5]

The paper discusses a number of mathematical areas in which a competing research programme was abandoned or areas that some mathematicians believe suffer from conceptual anomalies. Areas in need of exactly the same quasi-empirical testing that is used in physics. The paper analyzes the following areas in detail.

1. **Need for new characterizations of infinity**
   Since Cantor discovered infinity in the 19th century, a number of important results in mathematics and physics have utilized Cantor's characterization of infinity. Quantum physics began with Planck's calculation of black body radiation. The result required that the number of non-interacting harmonic oscillators be countable.[6] One of the most important problems of string theory is the proliferation of infinities in the multi-dimensions of string theories (Smolin, 278-280). Physics is in need of new infinity

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1. For example Smolin, L. *The Trouble with Physics*, 2006, 289-297.
2. Meyer, S. "Proposal to teach Lakatos-Feyerabend-Kuhn Philosophy of Science", 2004, URL: www.tdl.com/˜smeyer/docs/lfk-essay.dec22.pdf. The paper contains quotations to establish the unified philosophical research programme.
3. Lakatos established the quasi-empirical nature of mathematics in Lakatos, I. *Proofs and Refutations*, 1976.
4. Pickering, A. *Constructing Quarks*, 1984, especially 26-27.
5. Finsler, P. (Booth, D. and Ziegler, R. eds.) *Finsler set theory: Platonism and Circularity,*, 1996. For a philosophical discussion of Finsler's phenomenological rejection of meta-mathematics, see Breger, H. (Gillies, D. Ed.). *Revolutions in Mathematics,* 1992, 249-264. Finsler was a Goettingen trained mathematician that made contributions to general relativity.
6. Bohm, D. *Quantum Theory,* 1951, 15.

research programmes.

Starting in the 1930's there has been tension between algebraic characterizations of infinity (Goedel's integers as sentences) versus computational characterizations (Turing machine recursively enumerable sets). Since the two views can generate the same sets, a research programme that distinguishes the two characterizations has never existed (or died with the end of the Vienna Circle). There is a serious anomaly caused by the lack of progress on the P=NP problem. The immediate and universal acceptance of P=NP in the 1970s as the correct computational complexity research programme is the same as the rapid and universal acceptance of string theory in the 1980's (Smolin, ch. 8, 16, especially p. 116). The lack of progress in solving P=NP might imply that the current computational complexity research programme is degenerating. If there is an infinity between aleph 0 and aleph 1, it could be the number of non deterministic Turing machines and would prove P != NP. This would require a detailed proof.

Another way to see the need for competing infinity research programmes is related to the development of fast computers. Infinity is viewed as finite but unbounded instead of as a definable concept that can only be studied by algebraic (Goedelian) methods. There are current disagreements on the interpretation of a a 1956 letter from Goedel to Von Neumann. The computer science research programme claims Goedel was saying that computational complexity is just finite calculations, but Goedel distinguished the abstract concept of infinity from finite but unbounded computation.[7] Mathematics needs new physical interpretations of infinity to drive new mathematics research programmes.

2. **Acceptance of structural morphism based mathematical objects**
Another conceptual revolution was the establishment of mathematics as combinatorial group theory. Current mathematics studies the structural nature of mathematical objects and looks for various mappings between objects. This was not always the case. The Bruce Chandler and Wilhelm Magnus detailed study of the rise of combinatorial group theory describes the establishment of the structural research programme.[8] It is not clear what an alternative research programme would be, but there was at least some opposition to the morphism research programme in the late 1920s. The University of Vienna did not offer a chair to Emil Artin (see M. Schlick letter in the Vienna Circle Archive). Also, physicist Felix Bloch stated in his AHQP interview with Thomas Kuhn that he regretted using groups in the early 1930s (AHQP transcript, p. 34, par. 6). String theory is a structural set of theories so if Smolin's criticism of string theory is correct, new non structural axiomatizations may be needed for theoretical progress.

3. **Mathematics of Financial Option Pricing is Testable**
Option pricing models are modern mathematical creations that can in principal be tested. The original Black Scholes option pricing model was based on put call parity (conversions). It was later supplanted by the Robert Merton formalization. The original concept by Fischer Black was based on market equilibrium (from physical equilibrium). According to Emanuel Dreman, Fischer Black did not use or believe in mathematical formalism.[9] Testing the Black Scholes option pricing model is interesting because it is part of the fabric of thought (except for edge cases, it defines correct pricing) just as alternatives to Aristotelian Cosmology before the Keplerian revolution was unthinkable.

In conclusion, not only is mathematics testable in the same sense physics is testable, but as argued by LFK and more recently by Smolin, progress is mathematics and theoretical physics requires theory proliferation (new axiomatizations) and research programme competition.

7. URL: weblog.fortnow.com/2006/04/kurt-gdel-1906-1978.html, paragraph 3.
8. Chandler, B. and Magnus W. *The History of Combinatorial Group Theory: A Case Study in the History of Ideas,* 1982.
9. I am following Dreman's description of the development of option pricing models and his recollections of Fischer Black. See Dreman, E. *My Life as a Quant,* 2004, 143-173, especially 167,168.